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**USC**Viterbi  
School of Engineering

# Efficient Exploration of Cyber-Physical System Architectures Using Contracts and Subgraph Isomorphism

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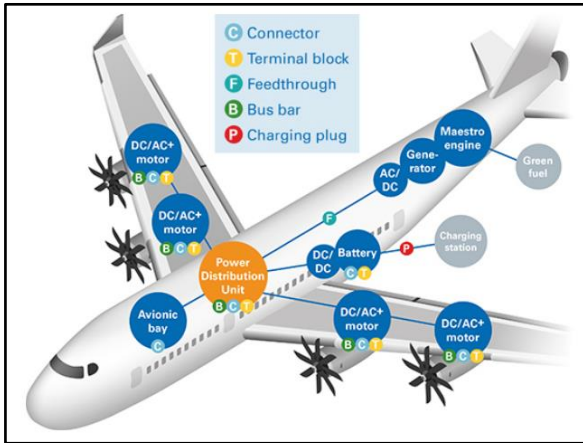
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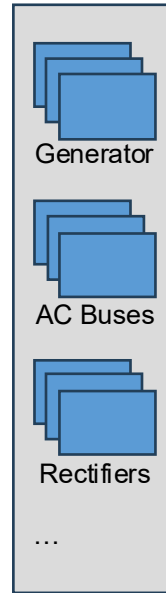
# The Challenge: Cyber-Physical System (CPS) Architecture Exploration

## Requirements

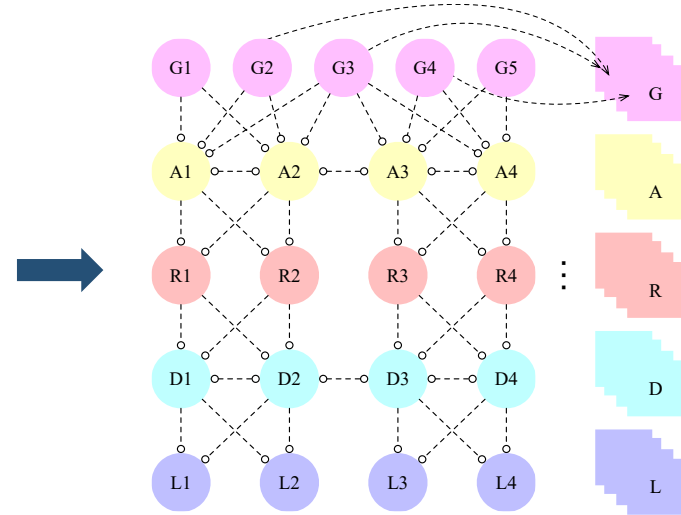
1. Timing
2. Energy Consumption
3. ...



Architecture: Set of components and interconnections



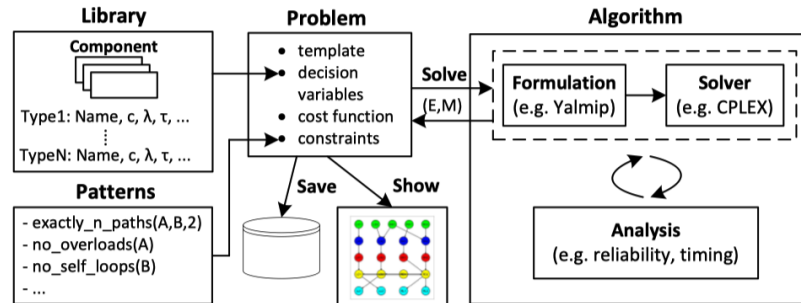
Library



Given a set of **components and connections**, **system requirements**, and an **implementation library**, find an **optimal architecture** that minimizes a cost function while satisfying all the requirements.

# CPS Design Space Exploration: Existing Approaches

- Satisfiability modulo theories [S. Peter et al., 2015]
- Graph-based methods: e.g., based on ordered binary decision diagram [H. Neema et al., 2014]
- **Mixed integer linear programming (MILP)**
  - **Flexible**: Can express multiple heterogeneous requirements and cost objectives
  - **ArchEx** [D. Kirov et al., 2017]: Proposes **efficient encodings** and **solutions strategies**
  - **Exponential complexity** with the size of the architecture



*Our Approach:* Support novel **decomposition strategies** and search **space pruning methods** to reduce exploration costs and enhance scalability

# Assume-Guarantee (A/G) Contracts Facilitate Compositional Reasoning

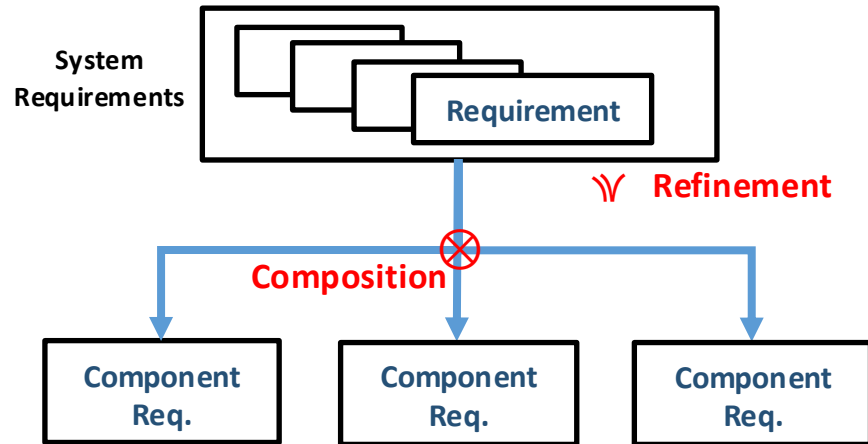
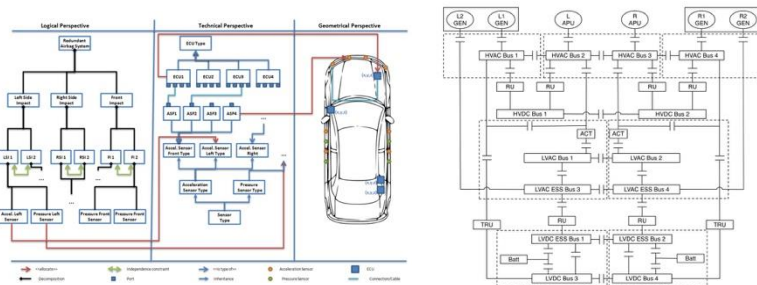
## Assume-Guarantee Contracts

[A. Benveniste, et al., 2012]

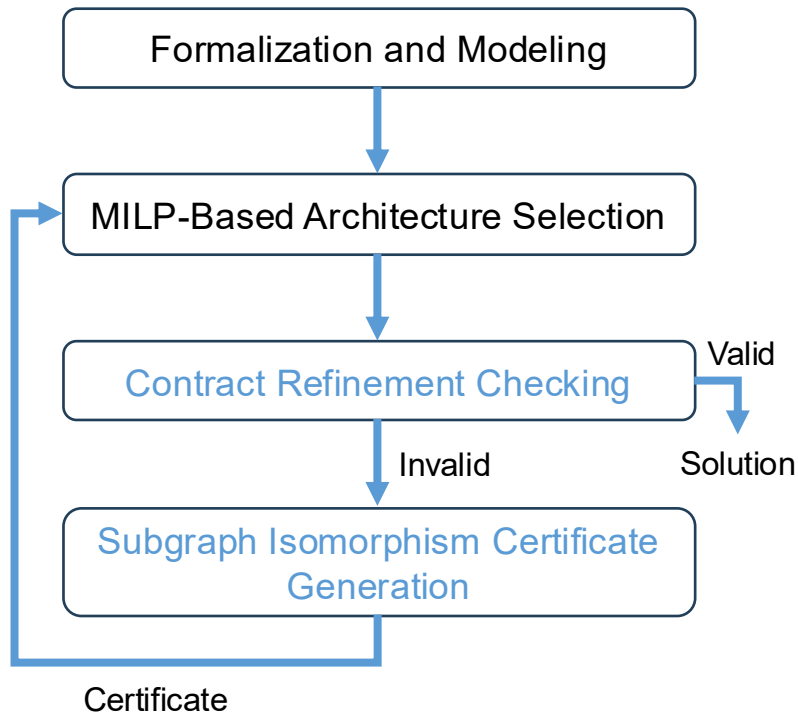
$$C = (A, G) \quad M \models C$$

Effectively applied to CPS design:

- Virtual integration testing and architecture design of a vehicle airbag system [Damm et al., 2011]
- Correct-by-construction aircraft electric power system design [Nuzzo et al., 2014]



# ContrArc: Contract-Based Architecture Exploration



- **Contract-based modeling and decomposition** methods to enhance scalability.
- **Coordination** between MILP solving and **graph analysis** to generate infeasible certificates
- Novel certificate generation method combining **contract refinement checking** with **subgraph isomorphism** to exclude invalid results

# Outline

- Contract-Based Formalization and Modeling
- MILP-Based Architecture Selection
- Contract Refinement Checking
- Subgraph Isomorphism Certificate Generation
- Case Studies

# Contract-Based Modeling and Formulation

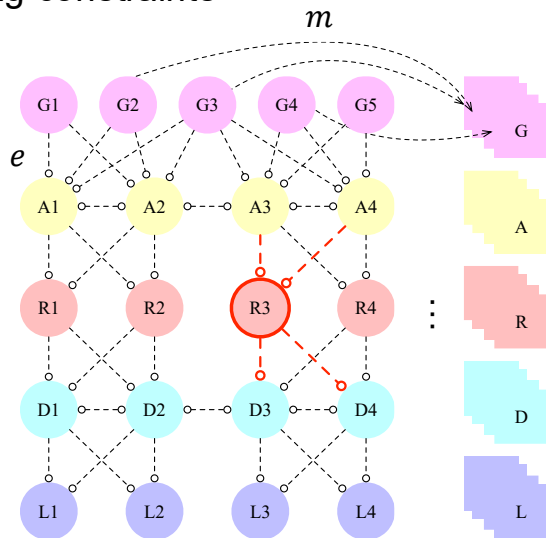
## Requirements:

### 1. Connection and mapping constraints

### 2. Flow constraints

### 3. Timing constraints

...



- *Assumptions*: if a component has connections, it can be mapped to an implementation in the library.
- *Guarantees*: if a component has input connections, it should have output connections.

$$C_{R3}^C = (\phi_{A_{R3}}^C, \phi_{G_{R3}}^C)$$

$$\phi_{A_{R3}}^C := \left( (e_{A3,R3} + e_{A4,R3} + e_{R3,D3} + e_{R3,D4}) \geq 1 \rightarrow \sum_i m_{R3,i} = 1 \right) \\ \wedge \left( (e_{A3,R3} + e_{A4,R3} + e_{R3,D3} + e_{R3,D4}) = 0 \rightarrow \sum_i m_{R3,i} = 0 \right)$$

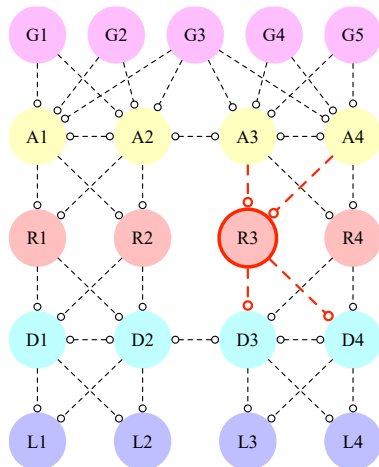
$$\phi_{G_{R3}}^C := \left( (e_{A3,R3} + e_{A4,R3}) \geq 1 \rightarrow (e_{R3,D3} + e_{R3,D4}) \geq 1 \right) \wedge \dots$$

# Formally Capture Requirements as Contracts

## Requirements:

1. Connection and mapping constraints
- 2. Flow constraints**
3. Timing constraints

...



### Component-level:

- *Assumptions:* Input flow remains below the prescribed throughput.
- *Guarantees:* Input flow and output flow must be balanced.

$$C_{R3}^F = (\phi_{A_{R3}}^F, \phi_{G_{R3}}^F)$$

$$\phi_{A_{R3}}^F := f_{R3}^P \geq e_{A3,R3} f_{A3,R3} + e_{A4,R3} f_{A4,R3} \geq f_{R3}^C$$

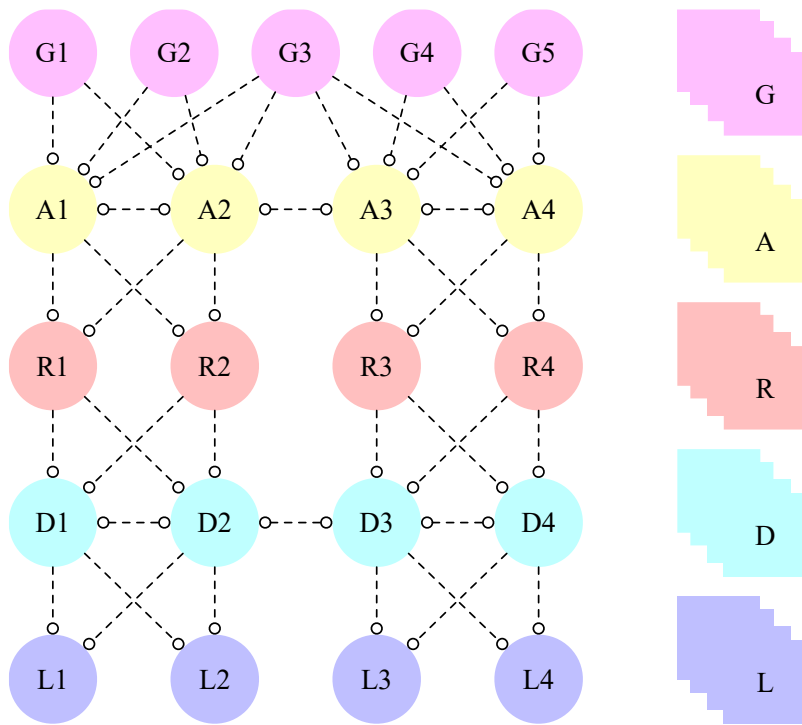
$$\phi_{G_{R3}}^F := e_{A3,R3} f_{A3,R3} + e_{A4,R3} f_{A4,R3} \geq e_{R3,D3} f_{R3,D3} + e_{R3,D4} f_{R3,D4} + f_{R3}^C$$

### System-level:

- *Assumptions:* The generated flow is bounded by the capacity of the source nodes.
- *Guarantees:* The total flow consumption is bounded.

# Select a Minimum-Cost Architecture Satisfying the Component-Level Contracts

$$\min_{e,m} \sum_{i=1}^N \alpha_i \beta_i c_i$$
$$\text{s.t. } \forall d, \bigwedge_{i=1}^N \phi_{A_i^d} \wedge \phi_{G_i^d}$$



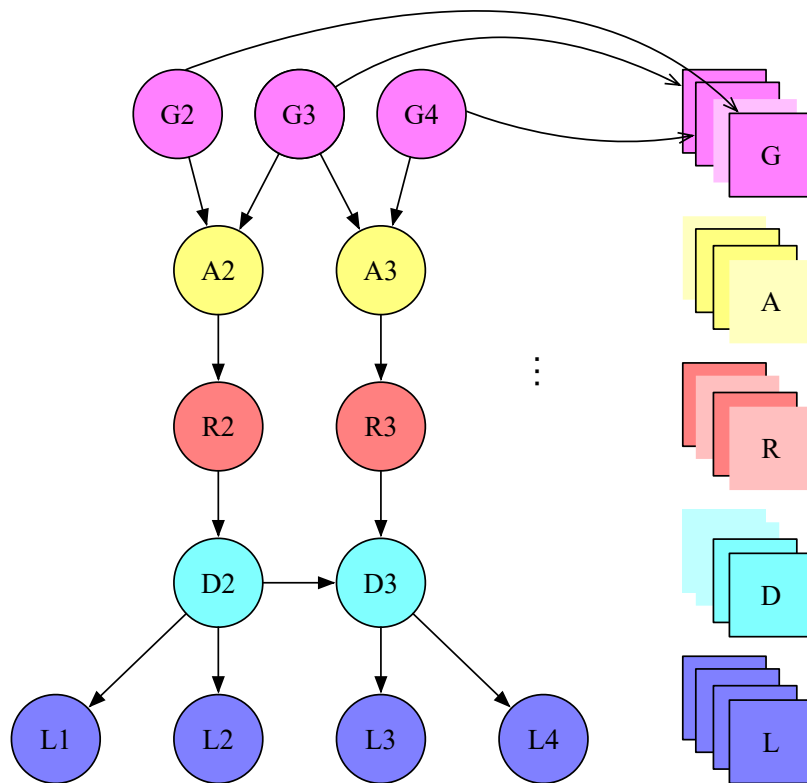
Network

Library

# Select a Minimum-Cost Architecture Satisfying the Component-Level Contracts

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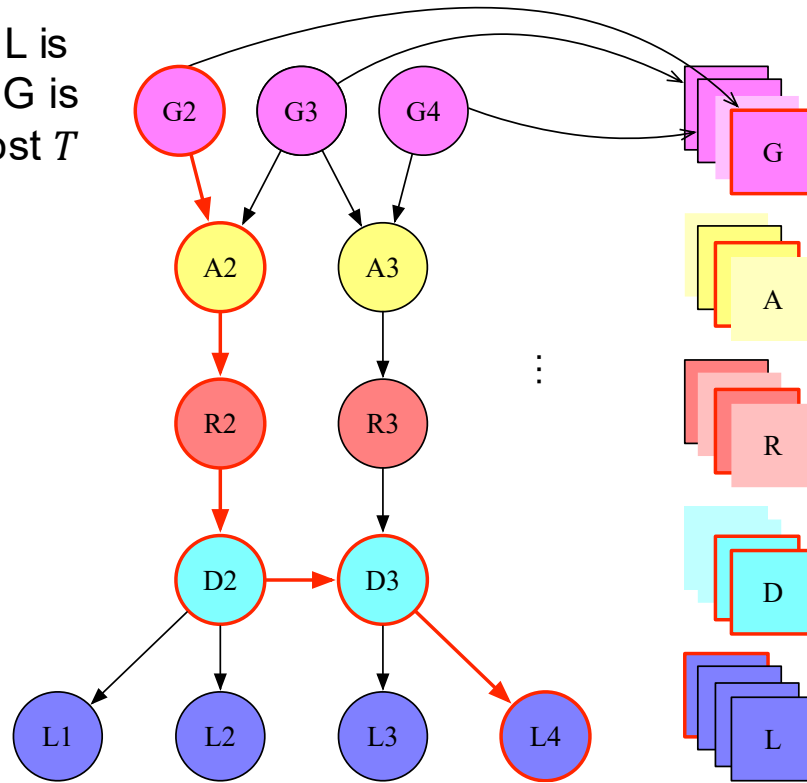
$$\text{s.t. } \forall d, \bigwedge_{i=1}^N \phi_{A_i^d} \wedge \phi_{G_i^d}$$



Selected Architecture

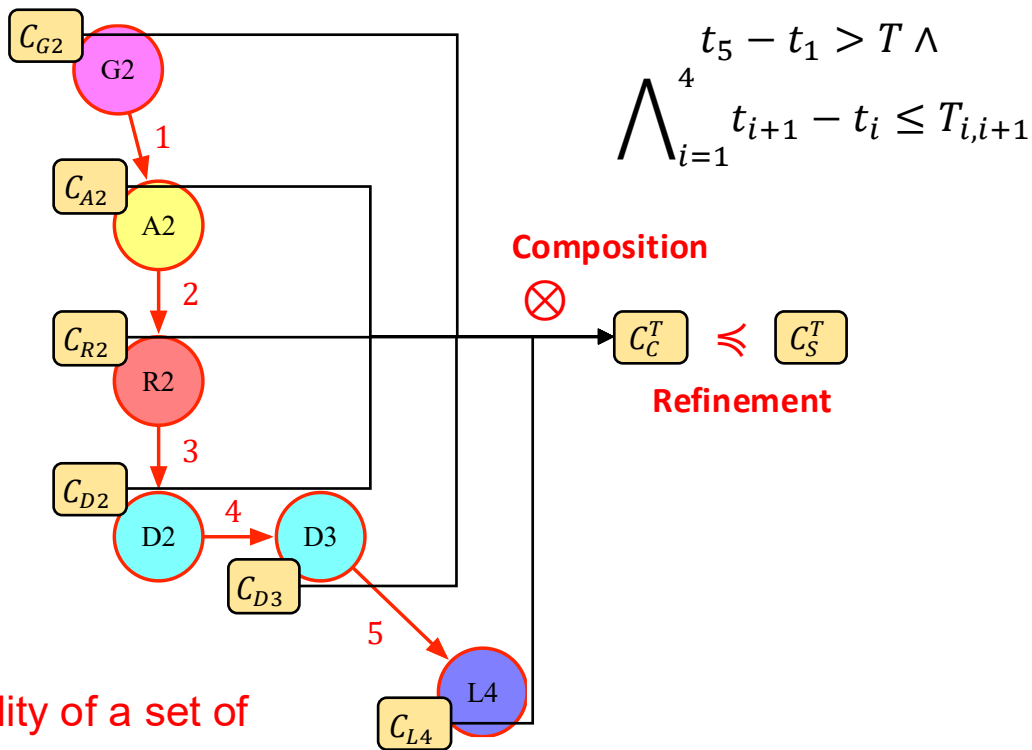
# Check Whether the Selected Architecture Satisfies the System-Level Contracts Via Refinement Checking

The delay by which a load  $L$  is powered after a generator  $G$  is switched on must be at most  $T$



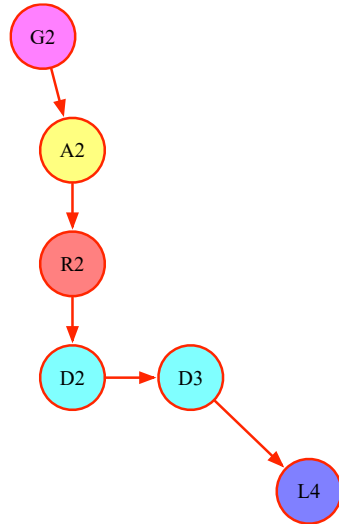
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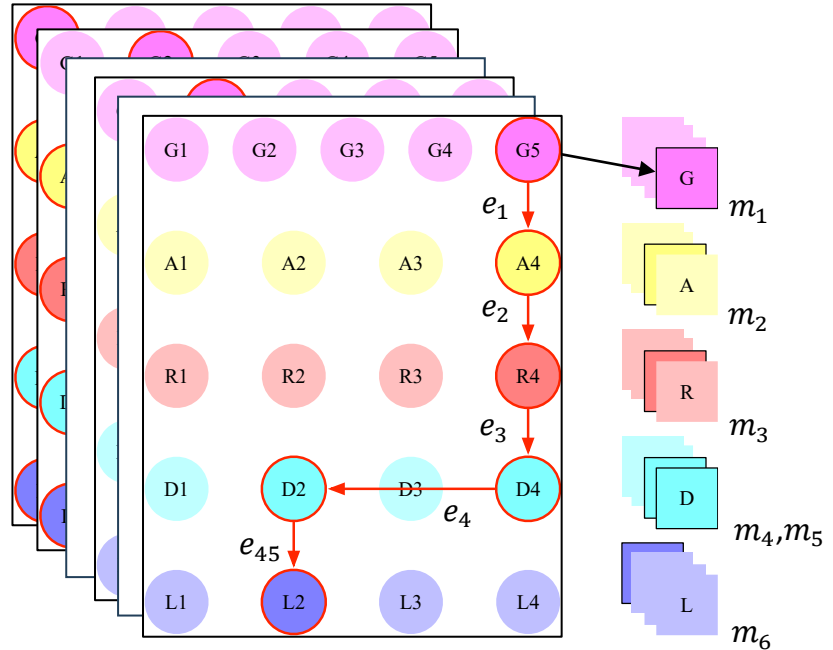


Reduced to checking the infeasibility of a set of mixed integer linear constraints

# Generate Infeasibility Certificates Via Subgraph Isomorphism Analysis



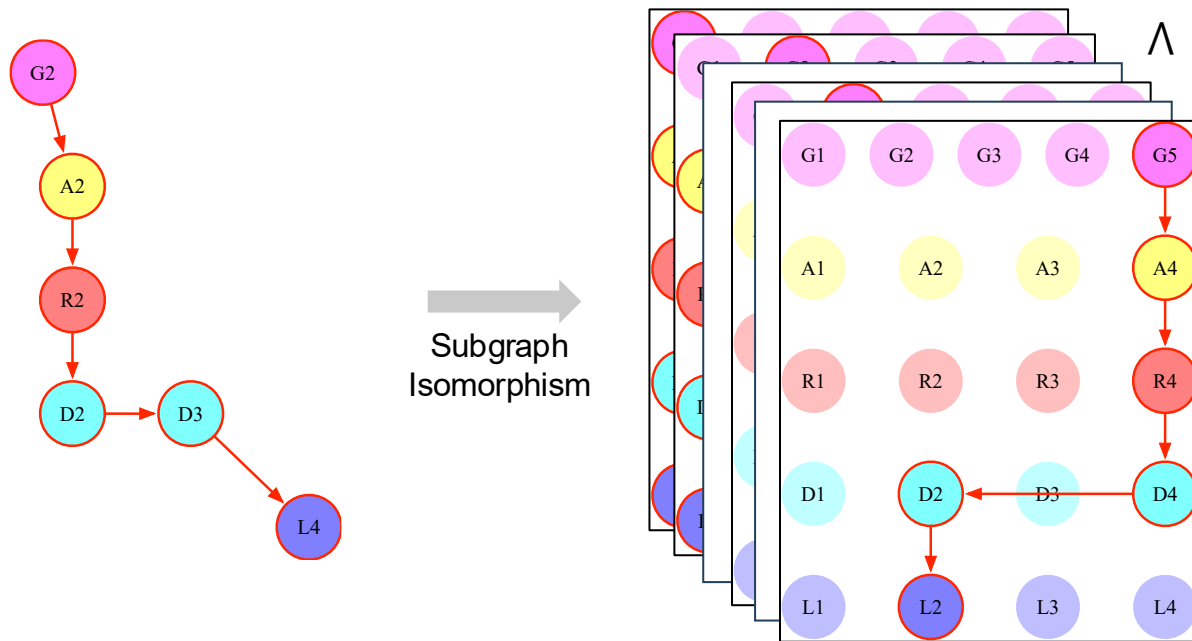
Subgraph Isomorphism



If a path is invalid, then the corresponding assignment to the edge and mapping variables should no longer be selected

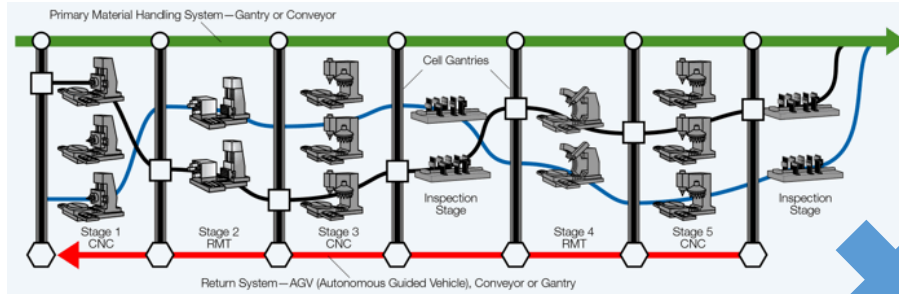
$$\Rightarrow \sum_i e_i + \sum_i m_i < 5 + 6$$

# Generate Infeasibility Certificates Via Subgraph Isomorphism Analysis

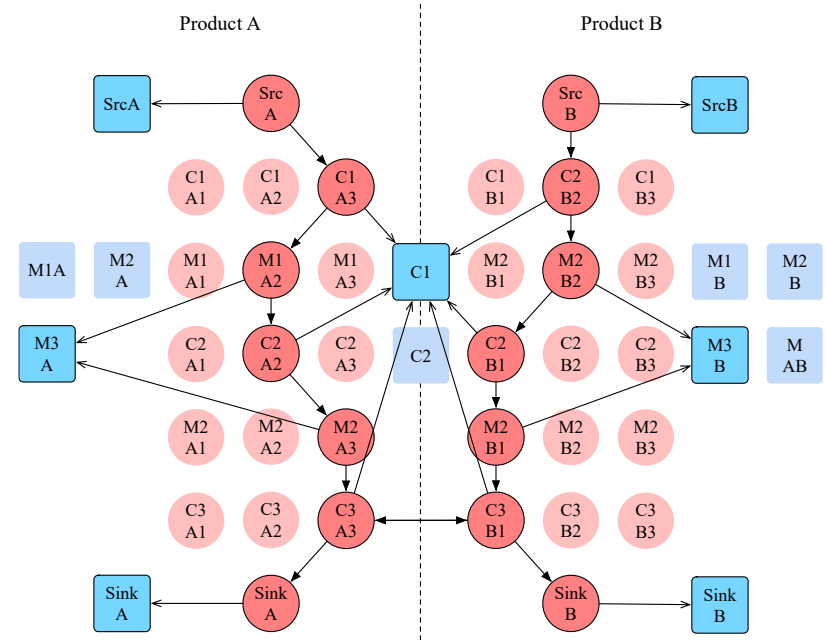


The constraints will be added to the **MILP-based problem** for architecture selection in the next iteration

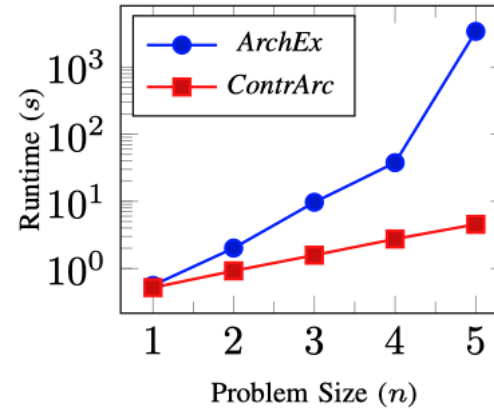
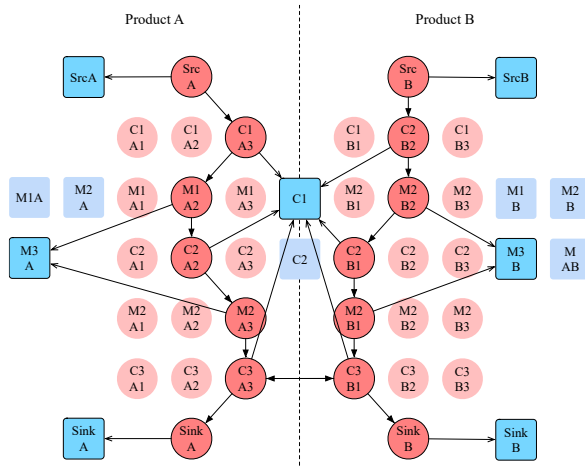
# Case Study: Reconfigurable Production Line (RPL)



- Components (in red): Source (Src), Machine (M), Conveyor (C), Sink
- Implementations (in blue)
- Assemble lines with the minimum cost such that the total delay to assemble a product is less than  $l$ , and the mass flow of product elements are balanced.

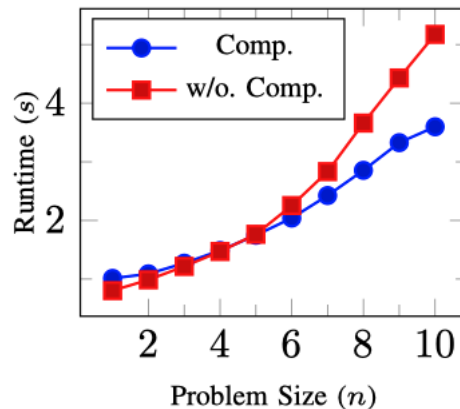
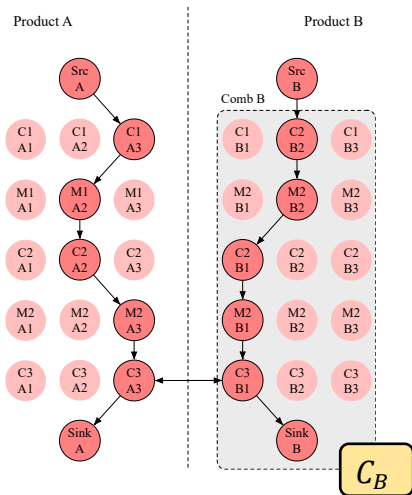


# Case Study: Reconfigurable Production Line (RPL)



- Up to **two orders of magnitude** acceleration with respect to **ArchEx** [K. Dmitrii, et al., 2017]

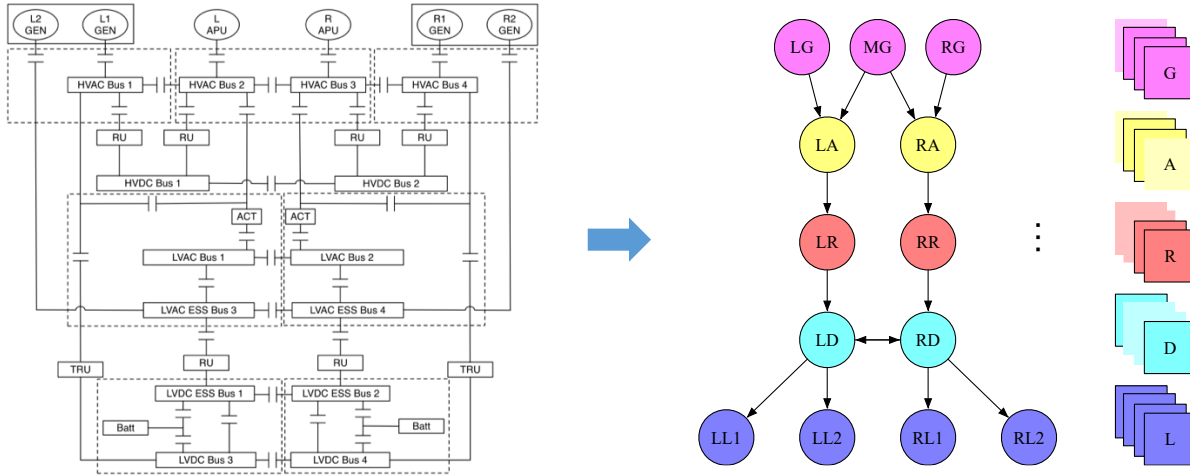
# Case Study: Reconfigurable Production Line (RPL)



- Up to **two orders of magnitude** acceleration with respect to **ArchEx** [K. Dmitrii, et al., 2017]
- **Compositional Exploration:** Partition the system and synthesize each line independently under appropriate assumptions

# Case Study: Electrical Power Network (EPN)

[Nuzzo et al., 2014]



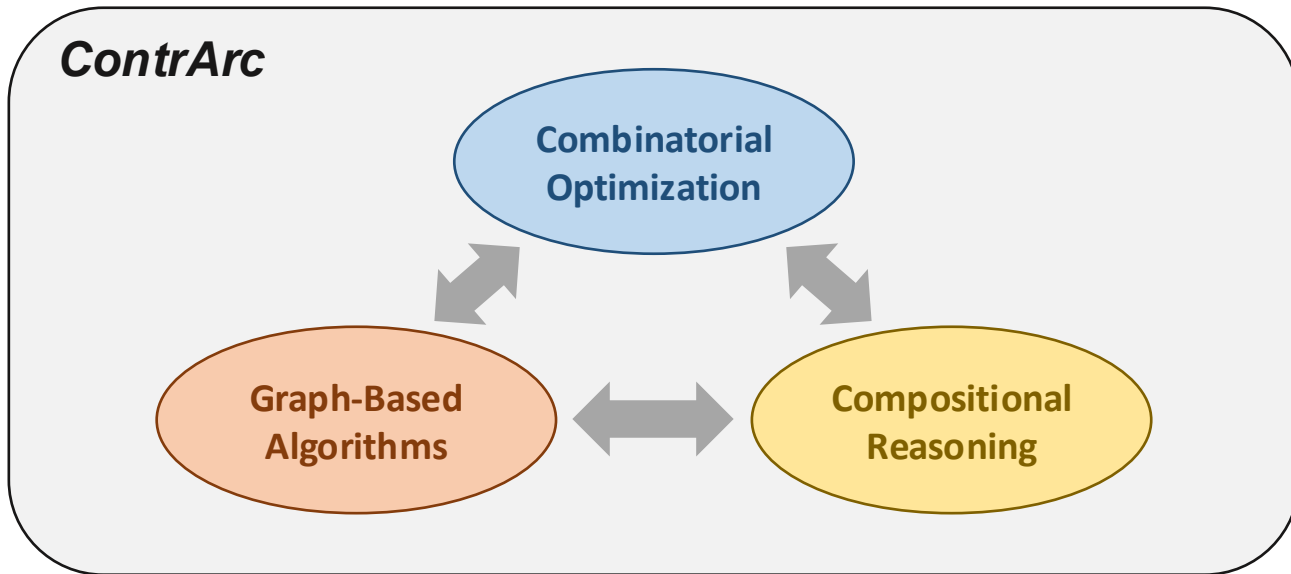
- Components (on left (L) and right(R) side): generator (GEN), AC bus, rectifier unit (RU), DC bus, load (L)
- Each component has four implementations
- Find the lowest cost EPN to power a set of critical loads, each requiring at least power  $p$ , with a delay less than  $l$

# Case Study: Electrical Power Network (EPN)

Max # in $\mathcal{T}$ ( $L, R, APU$ )	# of variables	# of constraints	w/o. decomposition		w/o. subgraph isomorphism		Complete ContrArc	
			Time (s)	# of iterations	Time (s)	# of iterations	Time (s)	# of iterations
1,0,0	454	195	0.57	3	0.58	3	0.56	3
2,0,0	1178	592	4.78	8	10.53	28	2.50	4
3,0,0	2280	1281	50.21	12	84.77	104	8.52	6
4,0,0	3868	2352	$6.31 \times 10^3$	18	$4.45 \times 10^3$	231	20.55	4
1,1,0	1138	576	11.18	22	10.72	24	9.15	24
2,1,0	2374	1383	$4.09 \times 10^3$	93	$4.82 \times 10^2$	320	27.12	20
2,2,0	4004	2508	$2.73 \times 10^4$	152	$5.59 \times 10^3$	1581	$1.55 \times 10^2$	34
1,1,1	1294	666	62.79	85	13.89	30	16.26	31
2,1,1	2604	1532	$1.57 \times 10^2$	56	$1.99 \times 10^2$	168	40.94	26
2,2,1	4320	2726	$2.35 \times 10^3$	60	$3.87 \times 10^3$	1353	$1.06 \times 10^2$	23
Average Ratio			$4.04 \times 10^3$	50.9	$1.07 \times 10^3$	384.1	38.67	17.5
			104.36	2.91	27.68	21.95	1.00	1.00

- **Compositional refinement checking** enables about **two orders of magnitude** acceleration
- **Subgraph isomorphism-based analysis** enables about **20 times less iterations** on average

# Conclusions



## Future Work:

- Extensions to support a broader set of requirements
- Integration of other graph-based algorithms with optimization methods