IJCAI 2025 MONTREAL

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Efficient Counterexample-Guided Fairness Verification and Repair of Neural Networks Using Satisfiability Modulo Convex Programming

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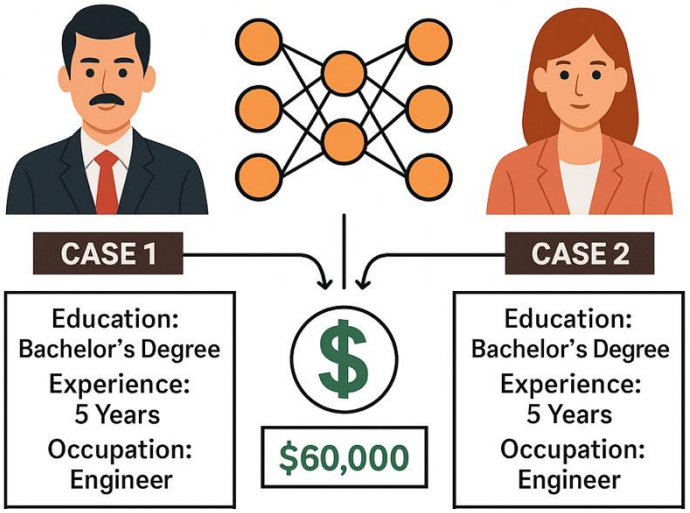
The Challenge: Ensuring Fair Decisions Made by Deep Neural Networks (DNNs)

- DNNs increasingly drive high-stakes decisions for which fairness is essential.

- ***Individual Fairness***: Individuals with similar unprotected attributes receive similar outcomes, regardless of their protected attributes
  - Unprotected attributes: qualifications, experience
  - Protected attributes: age, race



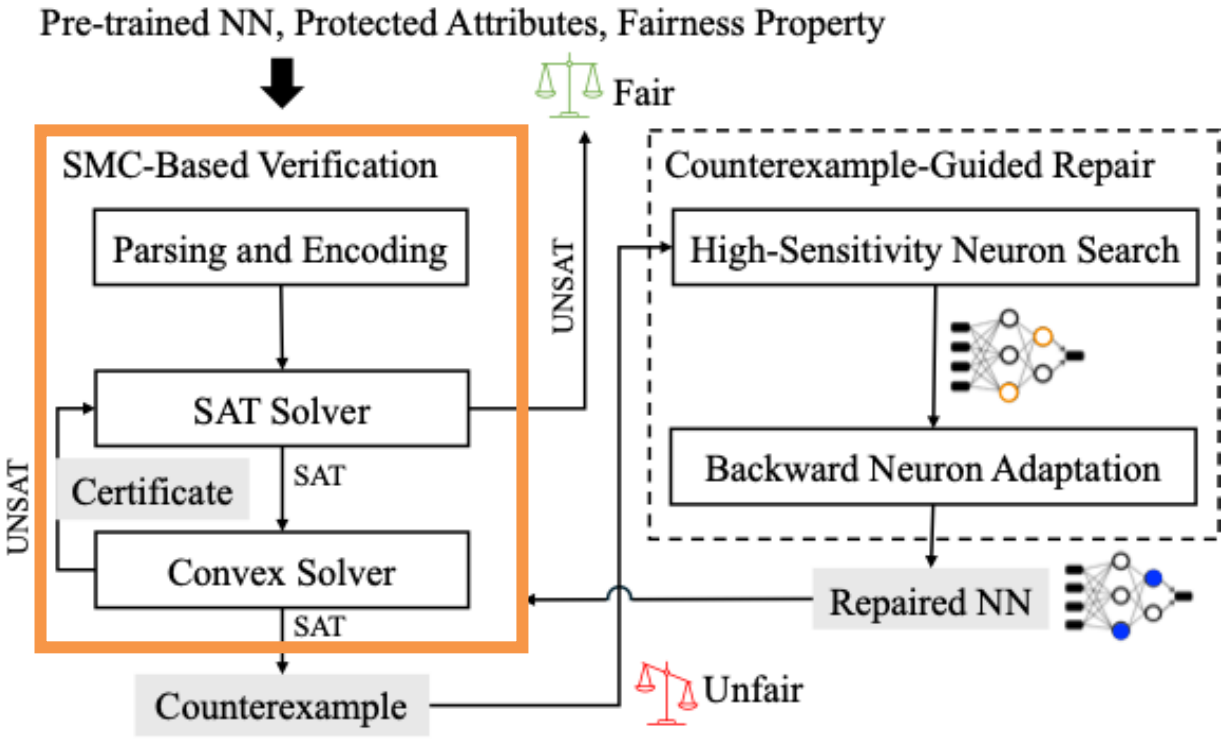
We need effective methods for fairness verification and repair

Fairness of Neural Networks: Existing Approaches

- Verification
  - Satisfiability modulo theories (SMT)-based methods [Benussi et al., 2022]
  - Mixed integer linear programming (MILP) [Biswas and Rajan, 2023; Mohammadi et al., 2023]
- Repair
  - Pre-processing: Remove bias from training data [Barocas et al., 2023]
  - In-processing: Modify model parameters during training [Dasu et al., 2024; Li et al., 2024; Gao et al., 2022; Fu et al., 2024]
  - Post-processing: Adjust model predictions after training [Nguyen et al., 2023; Li et al., 2023; Fu et al., 2024]

Our goal: More scalable verification and more efficient repair

FaVeR: Fairness Verification and Repair



Individual Fairness

- Instance: $\mathbf{x} = (x_1, x_2, \ldots, x_M)^T, \mathbf{x}' = (x_1', x_2', \ldots, x_M')^T$
- Attributes: $A = \{A_1, \ldots, A_M\}$; Protected attributes: $P \subset A$

Individual Fairness: No pair $(\mathbf{x}, \mathbf{x}')$ with

$$\forall \alpha \in A \backslash \mathrm{P} : x_a = x_a', \qquad \exists \beta \in P : x_\beta \neq x_\beta', \qquad f(\mathbf{x}) \neq f(\mathbf{x}')$$

Relax unprotected attributes

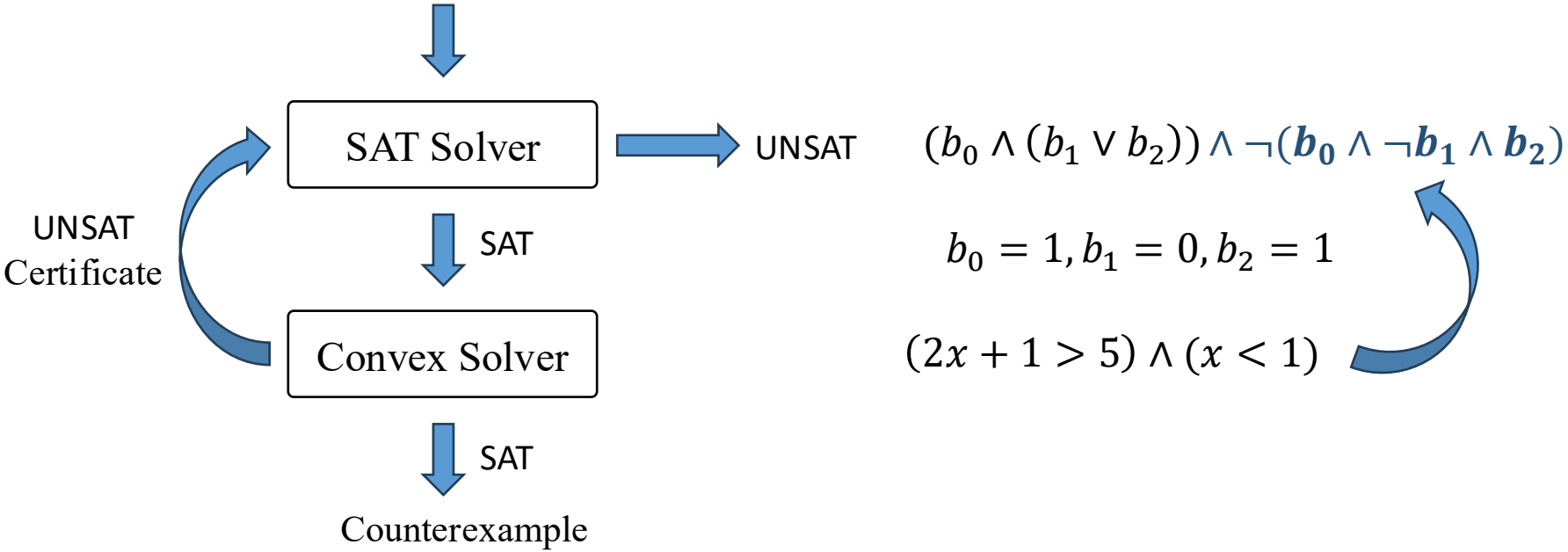
ϵ-Fairness: $|x_\alpha - x_\alpha'| \leq \epsilon_\alpha$

Verification: Check if $(\mathbf{x}, \mathbf{x}')$ exists with provided constraints

SMC-Based Verification

Check if x exists for $(2x + 1 > 5) \wedge ((x < 4) \vee (x < 1))$:

$$\big(b_0 \wedge (b_1 \vee b_2)\big) \wedge \big(b_0 \rightarrow (2x + 1 > 5)\big) \wedge \big(b_1 \rightarrow (x < 4)\big) \wedge (b_2 \rightarrow (x < 1))$$



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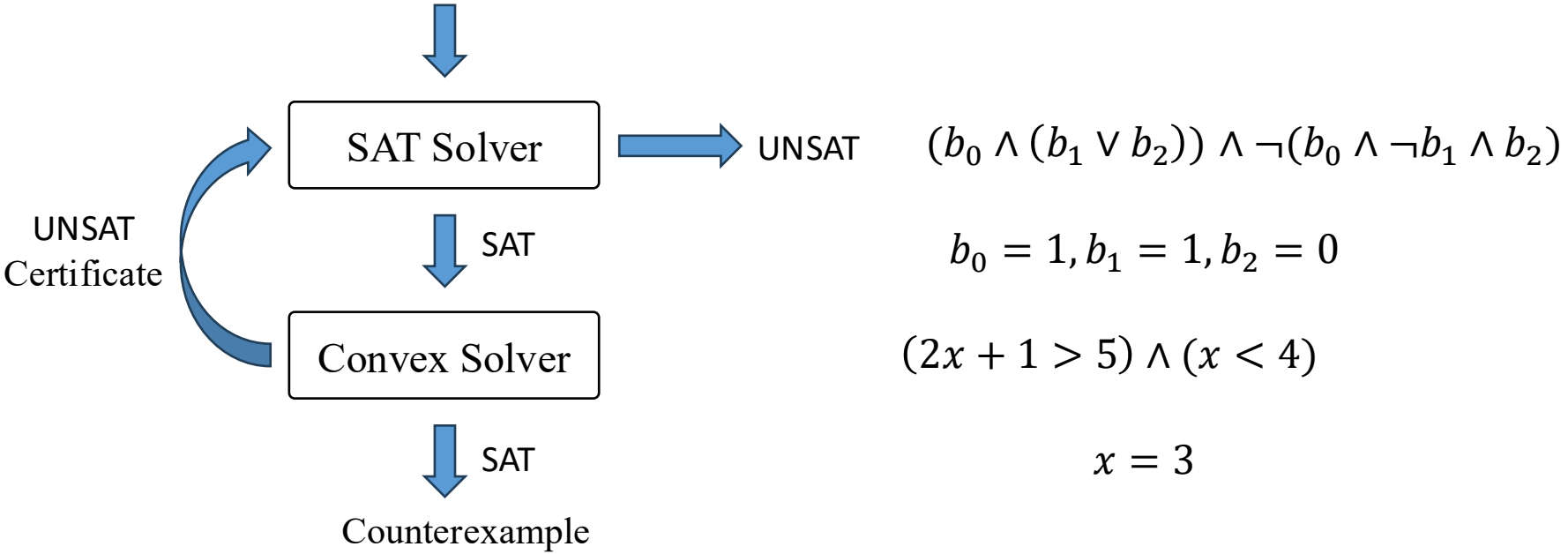
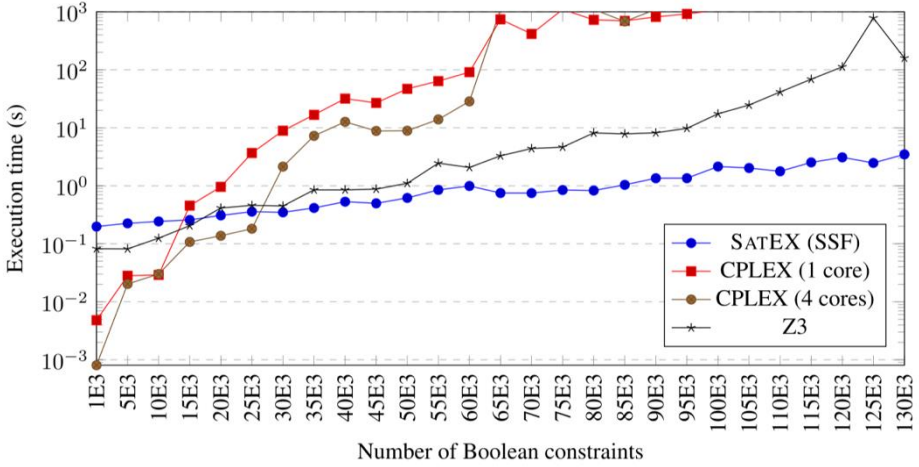
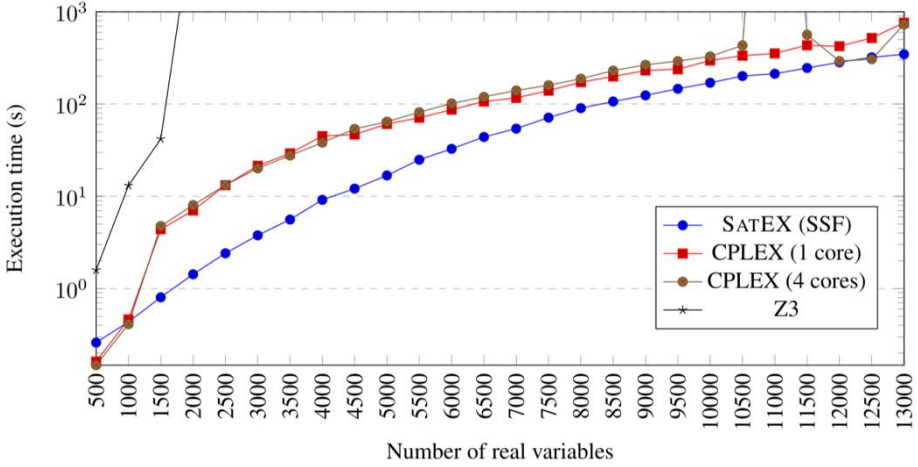


SMC-Based Verification





SMC [Shoukry et al., 2018] *is shown to outperform other methods for formulas with a large number of Boolean variables and convex constraints.*

Problem Encoding

ϵ-Fairness Property

$$\bigwedge_{A_\beta \in P} \left((m_\beta^{(0)} \rightarrow x_\beta = x'_\beta) \wedge (\neg m_\beta^{(0)} \rightarrow x_\beta \neq x'_\beta) \right) \wedge \left(\bigvee_{A_\beta \in P} m_\beta^{(0)} \right)$$

$$\bigwedge_{A_\alpha \in A \setminus P} \left((x_\alpha - x'_\alpha \leq \epsilon_\alpha^{(l)}) \wedge (x_\alpha - x'_\alpha \geq -\epsilon_\alpha^{(l)}) \right)$$

$$\left(m^{(L)} \rightarrow f(\mathbf{x}) > f(\mathbf{x}') \right) \wedge \left(\neg m^{(L)} \rightarrow f(\mathbf{x}) < f(\mathbf{x}') \right)$$

Feedforward Behavior

$$\left(m_i^{(l)} \rightarrow ((a_i^{(l)} \geq 0) \wedge (x_i^{(l)} = a_i^{(l)})) \right) \wedge \left(\neg m_i^{(l)} \rightarrow ((a_i^{(l)} < 0) \wedge (x_i^{(l)} = 0)) \right)$$

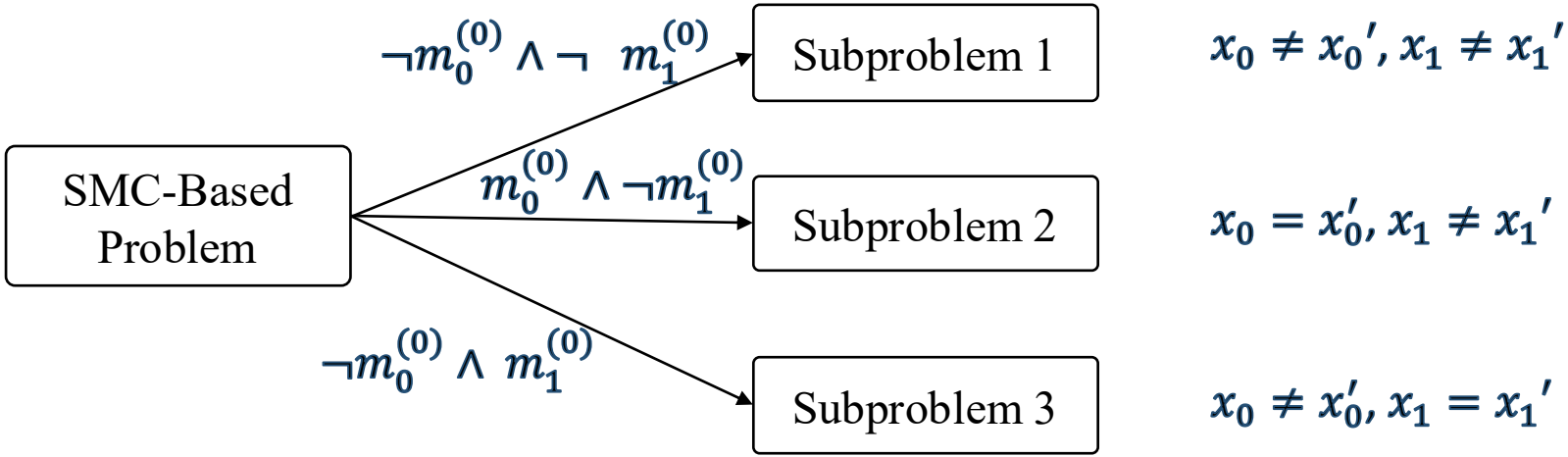
$$\bigwedge_{l=1}^{L-1} \left(\mathbf{x}^{(l)} = \phi(\mathbf{a}^{(l)}) \right) \wedge \left(\mathbf{x}^{(L)} = \mathbf{a}^{(L)} \right) \wedge \bigwedge_{l=1}^{L} \left(\mathbf{a}^{(l)} = \mathbf{W}^{(l,l-1)} \mathbf{x}^{(l-1)} + \mathbf{b}^{(l)} \right)$$

Boolean variables $\mathbf{m}$ are introduced to encode conditional branching behavior.

Problem Decomposition

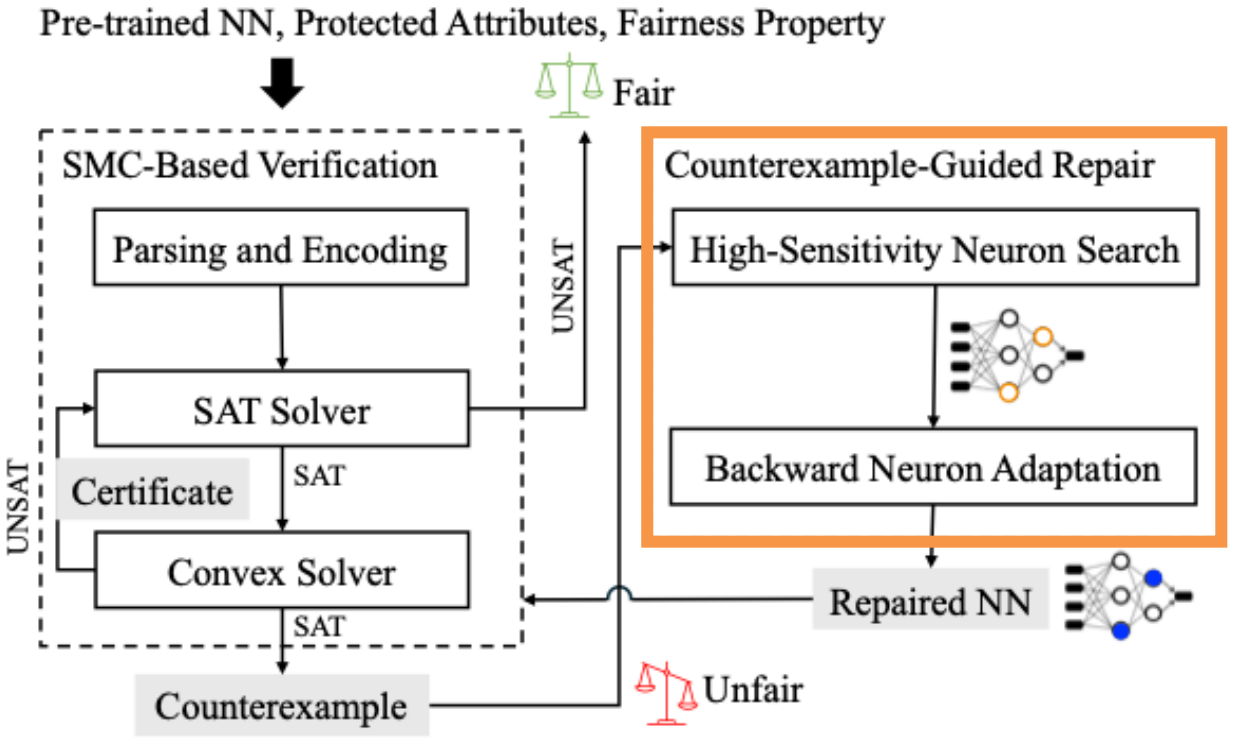
Introducing Boolean constraints enables decomposition of the verification problem.

Consider two protected attributes in the problem:



Add constraints in the SAT solver

FaVeR: Fairness Verification and Repair



High-Sensitivity Neuron Search

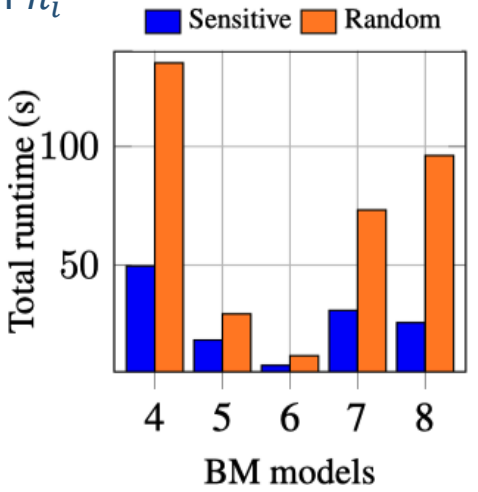
- Sensitivity score for neuron:

The activation of neuron n_i

$$S_i = |\sigma_i(\mathbf{x}) - \sigma_i(\mathbf{x}')|$$

Select high-sensitivity neurons with $S_i \geq \gamma$,

$$\gamma = \frac{1}{2}(\max_i S_i + \min_i S_i)$$



Search for the neurons with high contributions to unfairness

Backward Neuron Adaptation

Unfairness: $U = ||f(\mathbf{x}) - f(\mathbf{x}')||$

Update weights and bias of high-sensitivity neurons from layers L to 1:

$$W_{i,:}^{(l,l-1)} \leftarrow W_{i,:}^{(l,l-1)} + \Delta W_{i,:}^{(l,l-1)},$$

$$b_i^{(l)} \leftarrow b_i^{(l)} + \Delta b_i^{(l)},$$

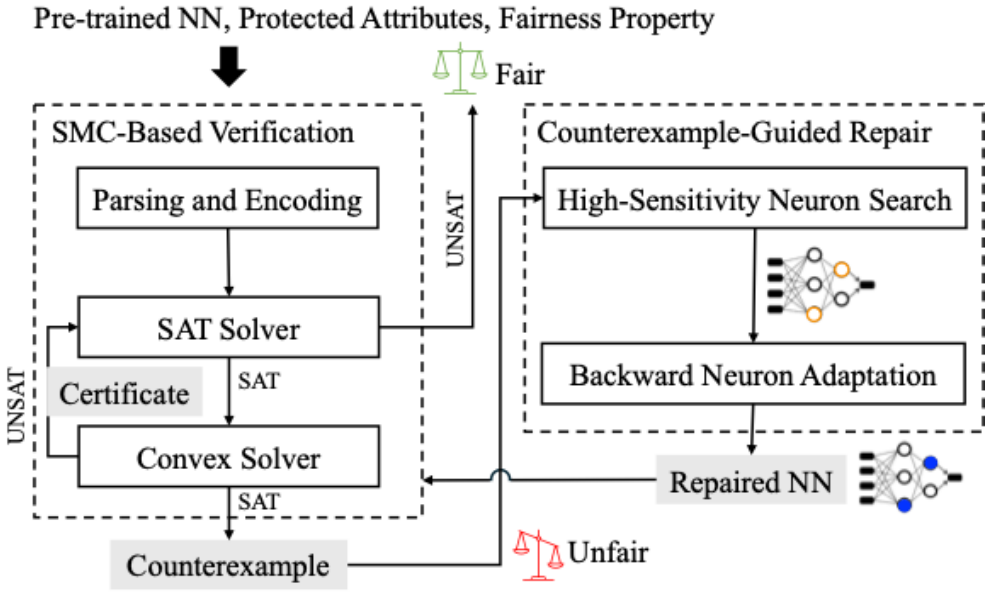
$$\Delta W_{i,:}^{(l,l-1)} = -\eta\, \lambda\, \mathrm{sign}\left(W_{i,:}^{(l,l-1)}\right) S_i^{(l)}\, W_{i,:}^{(l,l-1)},$$

$$\Delta b_i^{(l)} = -\eta\, \lambda\, \mathrm{sign}\left(b_i^{(l)}\right) S_i^{(l)}\, b_i^{(l)},$$

The same weight perturbation produces a larger shift in logits when applied to neurons closer to the output layer.

Each update reduces unfairness for small weights perturbations.

FaVeR: Fairness Verification and Repair



- Repair is rejected if accuracy drops below a specified threshold.
- The algorithm terminates when
  - The NN is fair, or
  - when all neurons have been adapted at most once.

Experiments: Fairness Verification

Benchmark: Compas (CP) [Kim et al., 2024]

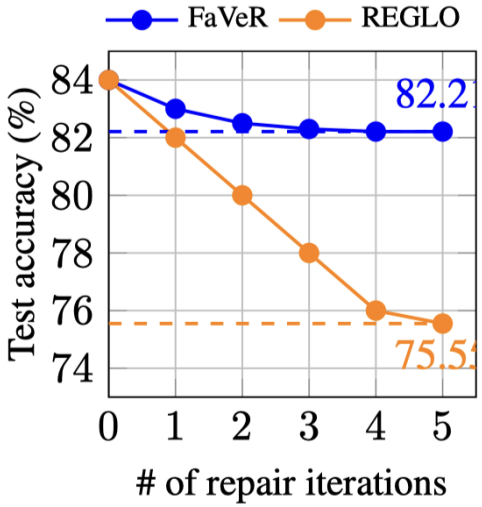
PA	*Model*	*#Layers*	*#Neurons*	*Fairify Ver.*	*Fairify Time(s)*	*FaVeR Ver.*	*FaVeR Time (s)*
Race	CP-1	2	24	SAT	27.11	SAT	**0.37**
	CP-2	5	124	SAT	63.24	SAT	**1.02**
	CP-3	3	600	**UNK**	1000+	UNSAT	**1.42**
	CP-4	4	900	**UNK**	1000+	SAT	**1.23**

FaVeR is faster and solves cases unsolved by state-of-the-art comparable approaches (Fairify, [Biswas and Rajan, 2023]*).*

Experiments: Repair for Fairness

Comparison with REGLO [Fu et al., 2024]

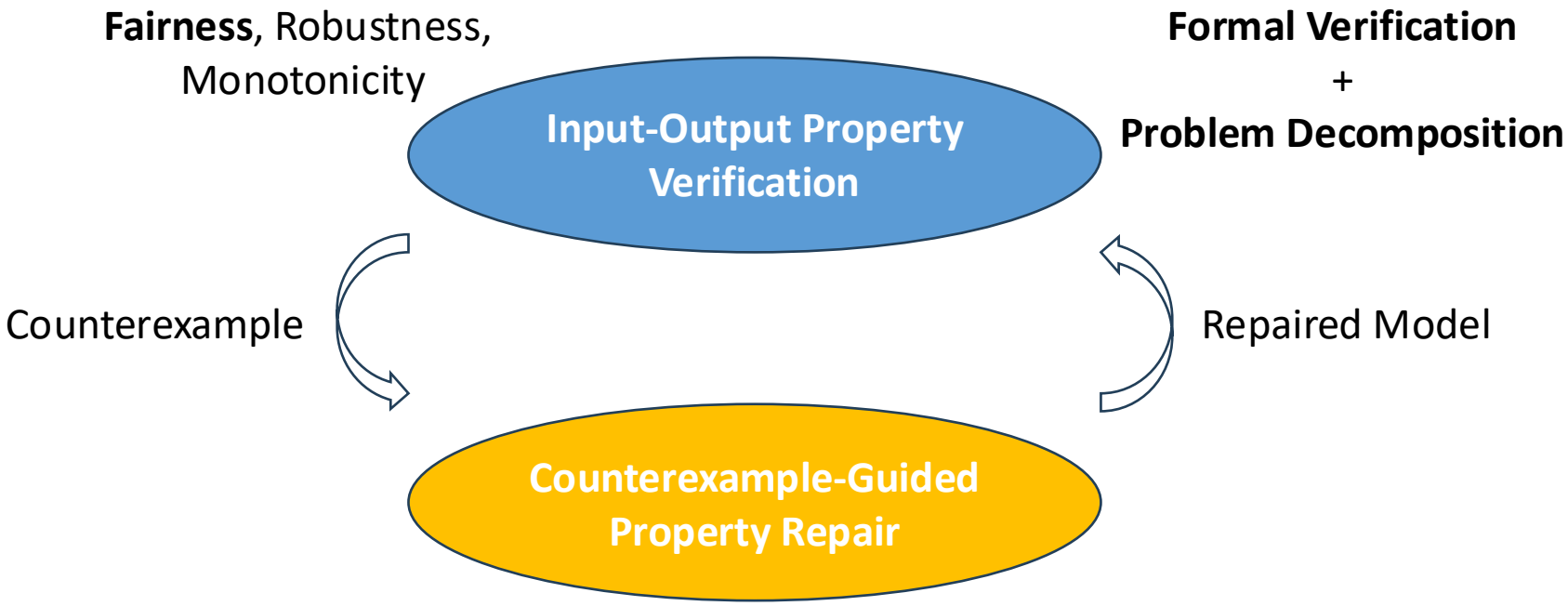
Benchmarks: Bank Marketing (BM), Adult Census (AC), German Credit (GC)



Model	Mean Initial Accuracy	FaVeR			REGLO		
		Mean Accuracy	Repair Rate	Mean Runtime	Mean Accuracy	Repair Rate	Mean Runtime
BM	88.14%	87.06%	100%	26.47 s	27.87%	60%	35.81 s
GC	71.60%	69.33%	100%	30.27 s	69.33%	100%	1.75 s
AC	82.33%	80.09%	100%	15.09 s	62.97%	100%	15.39 s

Efficient repair with less reduction in accuracy.

Conclusions



Localized adjustment, Constraint-augmented fine-tuning, …